

Pure Tone Scale vs. Equal Temperament Scale

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The world is tuned to $A = 440$ cps (cycles per second).

A piano is tuned to a Equal Temperament scale.

A cappella singing can be done to a Pure Tone scale.

Equal Temperament ... 12th root of 1.05946

$$440 * 1.05946 = 466.16$$

$$466.16 * 1.05946 = 493.88$$

note	chord	Equal Temperament scale (cps)	Pure Tone scale (cps)	difference (cps)	
A	root	110.00	110	0.00	same pitch
A	root	220.00	220	0.00	same pitch
E	fifth	329.63	330	0.37	sing sharp relative to piano
A	root	440.00	440	0.00	same pitch
A# / Bb		466.16			
B		493.88			
C		523.25			
C# / Db	third	554.37	550	-4.37	sing flat relative to piano
D		587.33			
D# / Eb		622.25			
E	fifth	659.26	660	0.74	sing sharp relative to piano
F		698.46			
F# / Db		739.99			
G	seventh	783.99	770	-13.99	sing flat relative to piano
G# / Ab		830.61			
A	root	880.00	880		same pitch

Overtones / undertones

$$\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta)$$

For 880 cps and 660 cps:

$$\sin 880 \sin 660 = \frac{1}{2} \cos(880 - 660) - \frac{1}{2} \cos(880 + 660)$$

$= \frac{1}{2} \cos(220) - \frac{1}{2} \cos(1540) \dots 220$ cps undertone, 1540 cps overtone

This is in reality more complex for singing as voices produce multiple harmonics.

If all are singing the Pure Tone scale very accurately, the sums and differences of the different harmonics and different voices/notes will reinforce each other.